

Achievable Rate and Energy Efficiency of Hybrid and Digital Beamforming Receivers with Low Resolution ADC

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Abstract—For 5G it will be important to leverage the available millimeter wave spectrum. To achieve an approximately omnidirectional coverage with a similar effective antenna aperture compared to state of the art cellular systems, an antenna array is required at both the mobile and basestation. Due to the large bandwidth and inefficient amplifiers available in CMOS for mmWave, the analog front-end of the receiver with a large number of antennas becomes especially power hungry. Two main solutions exist to reduce the power consumption: Hybrid BeamForming (HBF) and Digital BeamForming (DBF) with low resolution ADC. Hybrid beamforming can also be combined with low resolution ADCs. This paper compares the spectral and energy efficiency based on the chosen RF-frontend configuration. A channel with multipath propagation is used. In contrast to previous publication, we take the spatial correlation of the quantization noise into account. We show that the low resolution ADC are robust to small Automatic Gain Control (AGC) imperfections. We showed that in the low SNR regime the performance of DBF even with 1-2 bit resolution outperforms HBF. If we consider the relationship of spectral and energy efficiency then DBF with 3-5 bits resolution achieves the best ratio of spectral efficiency per power consumption of the RF receiver frontend over a very wide SNR region. The power consumption model is based on components reported in literature.

Index Terms—Wireless communication, millimeter Wave, low resolution ADC, hybrid beamforming.

I. INTRODUCTION

The use of the available bandwidth in the frequency range of 6 to 100 GHz is considered to be an essential part of the next generation mobile broadband standard 5G [1]. Due to the propagation condition, this technology is especially attractive for high data rate low range wireless communication. This frequency range is referred to as millimeter wave (mmW), even though it contains the lower centimeter wave range. In the last years, the available spectrum and the start of the availability of consumer grade systems lead to a huge increase in academic and industrial research. However, to fully leverage the spectrum while being power-efficient, the base band (BB) and radio frontend (RFE) capabilities must be drastically changed from state of the art cellular devices.

The use of high carrier frequencies above 6 GHz will go hand in hand with the implementation of large antenna arrays [1], [2]. The support of a large number of antennas at the

mobile and base station requires a new frontend designs. To attain a similar link budget, the effective antenna aperture of a mmWave system must be comparable to current systems operating at carrier frequencies below 6 GHz. Therefore, an antenna array at the base and mobile station is unavoidable. Since the antenna gain and thus the directivity increases with the aperture, an antenna array is the only solution to achieve a high effective aperture while maintaining an omnidirectional coverage.

A. Related Work

Current LTE systems have limited amount of antennas at the base and mobile stations. Since the bandwidth is relatively narrow, the power consumption of having a receiver RF chain with a high resolution ADC at each antenna is still feasible. For future mmWave mobile broadband systems, a much larger bandwidth [3] and a large number of antennas are being considered [1]. The survey [4] shows that ADCs with a large sampling frequency, and medium number of effective bits consume a considerable amount of power. The ADC can be considered as the bottleneck of the receiver [5].

The antenna array combined with the large bandwidth is a huge challenge for the hardware implementation, essentially the power consumption will limit the design space. At the moment, analog or hybrid beamforming are considered as a possible solution to reduce the power consumption. Analog or hybrid beamforming systems highly depend on the calibration of the analog components. Another major disadvantage is the large overhead associated with the alignment of the Tx and Rx beams of the base and mobile stations. Specifically, if a high gain is needed, the beamwidth is small, the acquisition and constant alignment of the optimal beams in a changing environment is very challenging [6], [7], [8].

The idea of hybrid beamforming is based on the concept of phase array antennas commonly used in radar application [9]. Due to the reduced power consumption, it is also seen as a possible solution for mmWave mobile broadband [10]. If the phase array approach is combined with digital beamforming the phase array approach might also be feasible for non-static or quasi static scenarios. In [11], it was shown that considering the inefficiency of mmWave amplifiers and the high insertion loss of RF phase shifters, it is better to perform the phase shifting in the baseband. The power consumption of both cases is comparable for these two cases, as long as the number of antennas per RF-chain remains relatively small.

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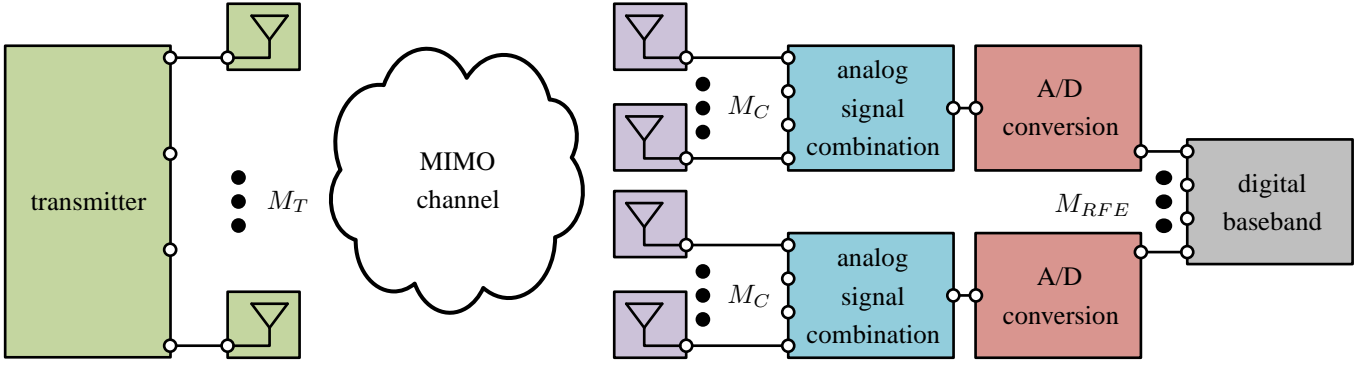


Fig. 1. System Model with M_T transmit antennas and M_C antennas at each of the M_{RFE} RF chains. Number of receive Antennas M_R is equal to $M_C \cdot M_{RFE}$.

Another option to reduce the power consumption while keeping the number of antennas the same is, to reduce the power consumption of the ADC by reducing the resolution. This can also be combined with hybrid beamforming. Some of these evaluations consider only the extreme case of 1-bit quantization [8], [12], [5]. In [13], [14] the A/D conversion is modeled as a linear stochastic process. Low resolution A/D conversion combined with OFDM in an uplink scenario are considered in [15], [16].

In [17], [18] hybrid beamforming with low resolution A/D conversion was considered. The energy efficiency / spectral efficiency trade-off of hybrid and digital beamforming with low resolution ADCs is assessed in [18]. But in contrast as shown in the system diagram in Figure 1, we consider an hybrid beamforming system that has exclusive antennas per RF-chain. In [18], a fully connected hybrid beamforming system is used, this has a large additional overhead associated with an increased number of phase shifter and larger power combiners. Also in this case additional amplifiers to compensate for the loss in the RF phase shifters and combiners are required. In [19], analog beamforming is compared with digital beamforming in terms of power efficiency.

B. Contribution

In this paper, we assess the achievable rate of hybrid and digital beamforming with low resolution A/D conversion in a multipath environment. The paper [18] showed that a digital beamforming system is always more energy efficient than a fully connected hybrid beamforming system. In contrast we use an hybrid beamforming system with exclusive antennas, which has a greatly reduced hardware complexity. Therefore in our evaluation the result is different compared to the previous evaluation.

- Achievable rate of hybrid and digital beamforming with low resolution ADC in a multipath environment is derived. The evaluation shows that the digital beamforming system for any resolution of the ADC always outperforms the hybrid system in the low SNR regime. It is important to stress that the low per antenna SNR regime is very likely the practical operating point of future mmWave systems. The low resolution ADC is essentially limiting

the performance in the high SNR regime. Therefore, an hybrid system with higher resolution ADC will always at some point surpass the digital system with lower resolution. We also show that small imperfections in the AGC do not influence the performance.

- Refinement of the Additive Quantization Noise Model (AQNM) by including the additional off-diagonal components of quantization-error covariance matrix. We show that for a scenario with every low resolution ADC (1-2 bit) and a larger number of receive antennas than transmit antennas, the performance including this off diagonal elements can substantially degrade the performance.
- Energy efficiency and spectral efficiency of the given systems are characterized. We show that for a wide SNR range the digital beamforming system is more energy efficient than the hybrid beamforming one. We also show that an A/D resolution in the range of 3-5 achieves the most energy efficient transmission.

Throughout the paper we use boldface lower and upper case letters to represent column vectors and matrices. The term $a_{m,l}$ is the element on row m and column l of matrix \mathbf{A} and a_m is the m th element of vector \mathbf{a} . The expressions \mathbf{A}^* , \mathbf{A}^T , \mathbf{A}^H , and \mathbf{A}^{-1} represent the complex conjugate, the transpose, the Hermitian, and the inverse of the matrix \mathbf{A} . The Discrete Fourier Transformation (DFT) $\mathcal{F}(\cdot)$ and its inverse $\mathcal{F}^{-1}(\cdot)$ and the Fourier transformation $\mathcal{F}\{\cdot\}$ and its inverse $\mathcal{F}^{-1}\{\cdot\}$ are also used.

II. SYSTEM MODEL

A. Signal Model

The signal model is shown in Figure 2. The symbols $\mathbf{x}[n]$, $\mathbf{H}[n]$, $\boldsymbol{\eta}[n]$, and $\mathbf{y}[n]$ represent the transmit signal, channel, noise, and receive signal of a system at time n . M_T transmit and M_R receive antennas are used. Since we assume a channel with multipath propagation the receive signal $\mathbf{y}[n]$ is defined as:

$$\mathbf{y}[n] = \sum_{l=0}^{L-1} \mathbf{H}[l] \mathbf{x}[n-l] + \boldsymbol{\eta}[n], \quad (1)$$

where L is the maximum delay of the channel in samples. The operation $F(\cdot)$ is defined as multiplication with the analog

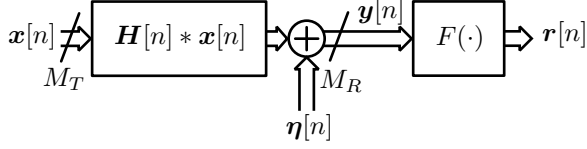


Fig. 2. Signal Model.

receiver beamforming matrix \mathbf{W}_R followed by a quantization operation $Q_b(\cdot)$ with resolution of b bits:

$$\mathbf{r}[n] = F(\mathbf{y}[n]) = Q_b(\mathbf{W}_R^H \mathbf{y}[n]). \quad (2)$$

The matrix \mathbf{W}_R has size $M_R \times M_{RFE}$. We restricted the system to have M_C antennas exclusively connected to one RF front-end chain (see Figure 1). Therefore the matrix \mathbf{W}_R has the form:

$$\mathbf{W}_R = \begin{bmatrix} \mathbf{w}_R^1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_R^2 & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{w}_R^{M_{RFE}} \end{bmatrix}, \quad (3)$$

where the vectors \mathbf{w}_R^i is the analog beamforming vector of the i th RF chain. We also restrict our evaluation to each RF chain utilizing the same number of antennas M_C . The vectors \mathbf{w}_R^i and $\mathbf{0}$ have length M_C .

The use of analog beamforming is envisioned in many future mobile broadband systems, especially in the mmWave frequency range ([20], [21]). Since complete channel matrix cannot be directly observed, one practical solution is scanning different spacial direction (beams) and then selects the configuration maximizing the SNR. There are many different possibilities for selecting the optimal beam, e.g. 802.11ad is using a procedure based on exhaustive search [22].

For the evaluation, we assume that the antennas of each RF chain form a Uniform Linear Array (ULA). If a planar wavefront is impinging on the ULA and the spacing of adjacent antennas is $d = \lambda/2$, the receive signal at adjacent antennas are phase shifted by $\phi_i = \pi \sin(\theta_i)$. The angle θ_i is the angle of planar wavefront relative to the antennas of the ULA. This formula assumes that a planar wavefront is impinging at the antenna array, and that the modulated signal is narrowband compared to the carrier frequency. With the constraint of observing only a single spatial direction, the receive vector \mathbf{w}_R^i for an ULA antenna array takes the form:

$$\mathbf{w}_R^i = [1, e^{j\phi_i}, e^{j2\phi_i}, \dots, e^{j(M_C-1)\phi_i}]^H. \quad (4)$$

In the special case of full digital beamforming ($M_C = 1$ and therefore $M_{RFE} = M_R$), \mathbf{W}_R is equal to the identity matrix \mathbf{I} of size $M_R \times M_R$.

The quantization operator $Q_b(\mathbf{a})$ is treating the I and Q component of each element of a vector \mathbf{a} separately. For a real valued, scalar input a , the output of the operation is defined as:

$$r = Q_b(a) = q^j \quad \forall \quad a \in [q_l^{j-1} q_l^j]. \quad (5)$$

Here q^j is the representative of the j th quantization bin with the input interval $[q_l^{j-1} q_l^j]$. To cover a real valued input the

left limit of the first interval q_l^0 and the right limit of the last interval $q_b^{N_q}$ are equal to $-\infty$ and ∞ . The number of quantization bins N_q is equal to 2^b . For real world ADC the difference between representatives of quantization bins q_r^j and the size of the quantization bins are uniform. We thus limit our evaluation to this set of quantizers. For the theoretical evaluation we assume Gaussian signaling. For the link level simulation we also assume that our receive signal can be approximated by a circular symmetric Gaussian. Consequently, we use the stepsize to minimize the distortion for Gaussian signals shown in [23].

Since the actual receive power at each antenna can be different, an AGC needs to adapt a Variable Gain Amplifier (VGA) to generate the minimal distortion. To simplify our model, we assume that the AGC is always perfectly adapting to the current situation. Since in practice an AGC can't accomplish this task without error we will show how an imperfect AGC does change the system performance dramatically. We model this by a relative error to the perfect gain value.

For the rest of the paper we define the SNR γ as:

$$\gamma = \frac{\mathbb{E} \left[\left\| \sum_{l=0}^{L-1} \mathbf{H}[l] \mathbf{x}[n-l] \right\|_2^2 \right]}{\mathbb{E}[\|\boldsymbol{\eta}[n]\|_2^2]}. \quad (6)$$

This formula is basically just describing the average SNR at each antenna. It is important to note that the expectation takes the realization of the channel and realizations of \mathbf{x} into account.

B. Channel Model

Dependent on the scenario, different channel models are used:

- Finite path model with path arriving at the same time
- Finite path model with exponential Power Delay Profile (PDP)

The channel models assumes different rays impinging on the receiver antenna array. In the first step, they are assumed to arrive at the receiver antennas at the same time. Under the assumption of an ULA at the transmitter and receiver, a channel consisting of K different rays can be modeled as:

$$\mathbf{H} = \frac{1}{\sqrt{KM_T}} \sum_{k=1}^K \alpha(k) \mathbf{a}_r(\phi_r(k)) \mathbf{a}_t^T(\phi_t(k)). \quad (7)$$

The vectors $\mathbf{a}_r(\phi_r(k))$ and $\mathbf{a}_t(\phi_t(k))$ are the arrays steering vectors at the receiver and transmitter. The phase shift between the signal of adjacent antenna elements $\phi_r(k)$ and $\phi_t(k)$ of path k depend on the angle of arrival $\theta_r(k)$ and departure $\theta_t(k)$.

$$\mathbf{a}_r^T(\phi_r(k)) = [1, e^{j\phi_r(k)}, e^{j2\phi_r(k)}, \dots, e^{j(M_r-1)\phi_r(k)}]. \quad (8)$$

The complex gains $\alpha(k)$ are circular symmetric Gaussian distributed with zero mean and unit variance. The angle of arrival $\theta_r(k)$ and departure $\theta_t(k)$ are uniform distributed in the range of $-\pi$ to π .

Since in real world scenario the different rays are reflection of different scatterers, the path of each of these rays from the

transmitter to the receiver has a different length. This results in different rays arriving at the receiver at different time. In a simplified case, it can be expected that the path that arrives at a later time have a lower power.

$$\mathbf{H}(l) = \frac{1}{\sqrt{M_T}} \alpha(l) \mathbf{a}_r(\phi_r(l)) \mathbf{a}_t^T(\phi_t(l)). \quad (9)$$

Here we assume, that at delay l only one ray arrives at the receiver. Here the complex gain of the ray $\alpha(l)$ is circular symmetric Gaussian distributed with zero mean and variance defined according to:

$$v_l = \mathbb{E} [|\alpha(l)|^2] = e^{-\beta l}. \quad (10)$$

The parameter β defines the how fast the power decays in relation to the sample time. The additional parameters are the maximum channel length in samples L and the number of actual present channel tabs P . This means that for the presence of all possible present channel rays \mathbf{v} of length L , P positions are selected for each channel realization. At all other positions, \mathbf{v} is equal to 0. To normalize the average power, the variance vector \mathbf{v} is normalized by:

$$\mathbf{v}_n = \frac{\mathbf{v}}{\|\mathbf{v}\|^2}. \quad (11)$$

C. Power consumption model

In a future 5G millimeter Wave mobile broadband system, it will be necessary to utilize large antenna arrays. Since the power consumptions scales about linear with number of antennas, it could also get very large. It is therefore important to compare the power consumption of different receiver architectures. In this section we make a power model for analog/hybrid beamforming to digital beamforming in combination with low resolution ADCs.

Since the spectrum in the 60 GHz band can be accessed without a license, it got significant attention. Especially the WiGig (802.11ad) standard operating in this band, increased the transceiver RF hardware R&D activities. Many chips were reported from industry and academia. Thus, it is safe to assume that the design reached a certain maturity, and performance figures derived from them represent the performance that is possible for a low cost CMOS implementation today.

According to the discussion in [24], baseband or IF phase shifting in contrast to RF phase shifting is assumed. This has the advantage of increased accuracy, decreased insertion loss, and reduced gain mismatch. In [24], the authors showed that the power consumption for a low number of antennas per RF-chain is equivalent to a system utilizing RF phase shifters.

All three systems utilize the same direct conversion receiver (Figure 3) to convert the signal into the analog baseband. For each system, we assume that the Local Oscillator (LO) is shared by the whole system. For the case of analog/hybrid beamforming systems, the analog baseband signals are phase shifted and then combined to generate the input signal of the M_{RFE} RF chains (Figure 4).

The A/D conversion consists of a VGA that is amplifying the signal to use the full dynamic range of the ADC (Figure 5). For the special case of 1-bit quantized digital beamforming

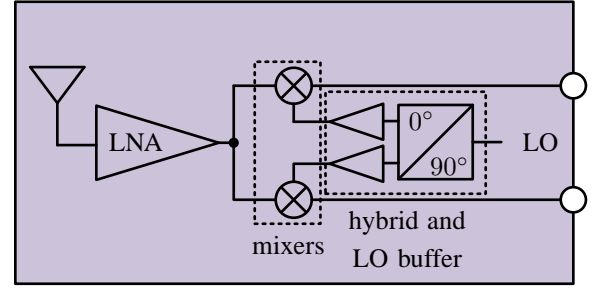


Fig. 3. Direct conversion receiver.

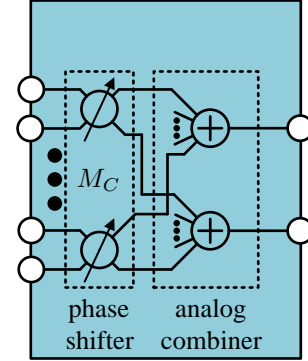


Fig. 4. Analog signal combination.

a VGA is not necessary. It can be replaced by a much simpler limiting amplifier (LA).

The power consumption of each component, including a reference, are shown in Table I. A LO with a power consumption as low as 22.5mW is reported in [25]. The power consumption of a LNA, a mixer including a quadrature-hybrid coupler, and a VGA are reported in [26] as 5.4, 0.5, and 2mW. The 90° hybrid and the clock buffer reported in [27] have a combined power consumption of 3 mW. The power consumption of the mixer reported in [28] is as low as 0.3mW. The survey in [4] also gives a good overview of state of the art ADCs. It shows the parameters effective number of bits (ENOB), sampling rate, and power consumption. Taking the predicted curve for the Walden FOM in [4] for a sampling frequency of 2.5 GS/s, we get 15 fJ per conversion step. A LA that consumes 0.8mW is reported in [29]. In the 1-bit quantized system, the LA (aka. Schmitt trigger) is already producing a digital signal, therefore the 1-bit ADC can be replaced by a flip flop (FF). The power consumption of a FF is negligible compared to the rest of the circuit.

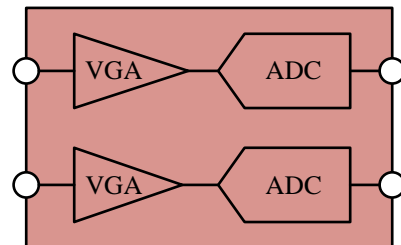


Fig. 5. A/D conversion.

TABLE I
COMPONENTS WITH POWER CONSUMPTION.

label	component	power consumption	reference
P_{LO}	LO	22.5mW	[25]
P_{LNA}	LNA	5.4mW	[26]
P_M	Mixer	0.3mW	[28]
P_H	90° hybrid and LO buffer	3mW	[27]
P_{LA}	LA	0.8mW	[29]
P_1	1-bit ADC	0mW	
P_{PS}	phase shifter (PS)	2mW	[11], [24]
P_{VGA}	VGA	2mW	[26]
P_{ADC}	ADC	$15 \frac{fJ}{\text{conv. step}} \times f_s 2^{ENOB} \text{W}$	[4] [30] [31]

With the power consumption of the components, it is possible to compute the power consumption of the overall receiver front-end P_R as:

$$P_R = P_{LO} + M_R (P_{LNA} + P_H + 2P_M) + \text{flag}_C (M_R P_{PS}) + M_{RFE} (-\text{flag}_{1\text{bit}} (2P_{VGA} + 2P_{ADC}) + \text{flag}_{1\text{bit}} (2P_{LA})), \quad (12)$$

where flag_C is the flag indicating if analog combining is used:

$$\text{flag}_C = \begin{cases} 0, & M_{RFE} = M_R \\ 1, & \text{else} \end{cases} \quad (13)$$

The flag $\text{flag}_{1\text{bit}}$ is indicating if 1 or multibit quantization is used. In the case of 1-bit quantization, the power consumption of the VGA is replace by the one of the LA and the power consumption of the 1-bit quantizer is neglected with the above stated reasoning. This formula now contains all special case of digital beamforming ($M_{RFE} = M_R$), analog beamforming ($M_R > 0$ and $M_{RFE} = 1$) and hybrid beamforming for the configurations in between those extreme cases.

A receiver directly designed for the 1-bit quantization digital beamforming systems is very likely to improve the power consumptions even further. Due to the 1-bit quantization at the end of the receiver, the linearity required of the circuits before is greatly reduced. This would enable specialized designs to improve the performance in terms of power consumption.

III. ACHIEVABLE RATE EXPRESSIONS

In this subsection achievable rate expressions for different scenarios are derived. The different scenarios are any combination of flat fading channel, multipath channel, hybrid beamforming and digital beamforming with low resolution A/D conversion. In the case of hybrid beamforming, first the analog receive vectors are calculated. Afterwards, the system including the analog combiners are used as the input of a digital beamforming system.

A. Hybrid Beamforming Vectors

In the signal model, we already restricted the receive vectors \mathbf{w}_R^i of the i th RF chain to mimic the behavior of a spatial scan.

A practical system would have a set of predefined beamforming configuration that is scanned at every sub-arrays. To obtain the optimal results all combination of beams need to be tested by the receiver. This is a combinatorial problem with the size growing exponentially with the number of receiver RF-chains. To make the problem feasible, the scan is performed separately for each receiver RF chain. This problem can be formulated as:

$$\mathbf{w}_R^i(\hat{\phi}) = \arg \max_{\mathbf{w}_R^i(\phi_B)} \sum_{l=0}^{L-1} \|\mathbf{w}_R^i(\phi_B)^H \mathbf{H}^i(l)\|_2^2 \text{ with } \phi_B \in \mathbb{B}, \quad (14)$$

with \mathbb{B} being the set of all spatial direction ϕ_B that are scanned. The channel $\mathbf{H}^i(l)$ contains the M_C rows of $\mathbf{H}(l)$ that belong to the antennas of the i th RF chain.

For an actual system, this process would be like every antenna or RF chain is transmitting a predefined reference sequence that is orthogonal to all others. These transmission sequences has to be repeated till all Rx beamformer configurations are tested. The receiver would detect the sequence and estimate the channel for each Rx beamformer configuration transmitted sequence separately for each RF chain. Afterwards, the one with the configuration with the highest combined signal Power on each RF chain is selected and the data transmission begins.

To select the values in \mathbb{B} , we first calculated the array factor of the antenna array. With this array factor, we than select the spacing of the values ϕ_B uniform from 0 to 2π . Here we assume isotropic minimum scattering antennas. For ULA with spacing $\lambda/2$, the absolute value of the normalized array factor is defined as [32]:

$$AF = \frac{1}{M_C} \left| \frac{\sin(M_C \frac{\pi}{2} \sin(\theta - \phi_B))}{\sin(\frac{\pi}{2} \sin(\theta - \phi_B))} \right|. \quad (15)$$

That means that for actual arriving angle θ choosing $\phi_B = \theta$ is optimal. But this would mean that we have a infinite grid of ϕ_B . Assuming a single wavefront arriving at the receiver and an uniform distributed angle of the arriving signal θ , we get the following expression for the average error ϵ :

$$\frac{2}{\Delta} \int_0^{\frac{\Delta}{2}} 1 - \frac{1}{M_C} \left| \frac{\sin(M_C \frac{\pi}{2} \sin(x))}{\sin(\frac{\pi}{2} \sin(x))} \right| dx \leq \epsilon. \quad (16)$$

Setting a maximum allowed ϵ , we can solve the equation for the distance Δ between two angles in the set \mathbb{B} :

$$\frac{2}{\Delta} \left(\int_0^{\frac{\Delta}{2}} 1 - \frac{1}{M_C} \left| \frac{\sin(M_C \frac{\pi}{2} \sin(x))}{\sin(\frac{\pi}{2} \sin(x))} \right| dx \right) - \epsilon = 0. \quad (17)$$

A table for some configurations and the minimum number of elements in \mathbb{B} are shown Table II. It can be observed that for the given parameters, the minimum number of elements can be well approximated by $4M_C$. Therefore we select $4M_C$ elements uniform in the range from 0 to 2π to represent the set \mathbb{B} .

After selecting all beamforming vectors \mathbf{w}_R^i , the overall matrix \mathbf{W}_R is constructed. With \mathbf{W}_R , we can generate the

TABLE II
MINIMUM NUMBER OF BEAMS NECESSARY TO ACHIEVE MEAN ERROR
 $\epsilon = 10\%$.

M_C	minimum number of beams	approximation $4M_C$
2	7	8
4	14	16
8	29	32
16	58	64
32	115	128

effective channel $\mathbf{H}_C(l)$:

$$\mathbf{H}_C(l) = \mathbf{W}_R \mathbf{H}(l). \quad (18)$$

and the effective noise covariance matrix $\mathbf{R}_{\eta_C \eta_C}$:

$$\mathbf{R}_{\eta_C \eta_C} = \mathbf{W}_R \mathbf{R}_{\eta \eta} \mathbf{W}_R^H. \quad (19)$$

The effective channel and noise covariance matrix are then the input to the digital system with low resolution A/D conversion. Algorithm 1 shows the procedure of finding the receiver beamforming vectors \mathbf{w}_R^i given the channel and the number of antennas per RF chain.

Algorithm 1 Selection of the beamforming vectors.

Require: $\mathbf{H}(l)$, M_{RFE} and M_C

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1:  $\mathbb{B} \leftarrow \{\phi_1, \phi_2, \dots, \phi_{4M_C}\}$ 
2: for  $i \leftarrow 1$  to  $M_{RFE}$  do
3:   for  $j \leftarrow 1$  to  $4M_C$  do
4:      $\mathbf{w}_{tst} \leftarrow [1, e^{j\phi_1}, \dots, e^{j\phi_{4M_C}}]^H$ 
5:      $p^i(j) \leftarrow \sum_{l=0}^{L-1} \|\mathbf{w}_{tst}^H \mathbf{H}^i(l)\|_2^2$ 
6:   end for
7:    $\hat{j} \leftarrow \underset{j}{\operatorname{argmax}} p^i(j)$ 
8:    $\mathbf{w}_R^i \leftarrow [1, e^{j\phi_1}, \dots, e^{j\phi_{4M_C}}]^H$ 
9: end for
10: return  $\mathbf{w}_R^i \forall i = \{1, \dots, M_{RFE}\}$ 
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B. Modeling the quantization

As the model used in [13], [19], [17], we use the Busgang theorem to decompose the signal after quantization in a signal component and a uncorrelated quantization error e :

$$\mathbf{r}[n] = \mathbf{F} \mathbf{y}_C[n] + e[n], \quad (20)$$

with $\mathbf{y}_C[n]$ being the signal after the analog combining at the receiver. This is basically modeling the deterministic process of quantization as a random process. The quantization distortion factor $\rho(Q_b(\cdot))$ is defined as:

$$\rho(Q_b(\cdot)) = \mathbb{E} [|a - Q_b(a)|^2]. \quad (21)$$

The input of the quantizer is assumed to be zero mean unit variance Gaussian random variable a . The distortion factor $\rho(Q_b(\cdot))$ depends on the actual quantization operator $Q_b(\cdot)$

and represents the variance of the introduced distortion. As shown in [13], the matrix \mathbf{F} can be calculated as:

$$\mathbf{F} = \mathbf{R}_{\mathbf{r} \mathbf{y}_C} \mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}^{-1}. \quad (22)$$

With the definition of the distortion factor, this can be reduced to:

$$\mathbf{R}_{\mathbf{r} \mathbf{y}_C} = \mathbf{R}_{\mathbf{y}_C \mathbf{r}} = (1 - \rho(Q_b(\cdot))) \operatorname{diag}(\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C})^{-\frac{1}{2}} \mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}. \quad (23)$$

Plugging (23) into (22) results in:

$$\mathbf{F} = (1 - \rho(Q_b(\cdot))) \operatorname{diag}(\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C})^{-\frac{1}{2}}. \quad (24)$$

The covariance matrix \mathbf{R}_{ee} can be calculated from:

$$\mathbf{R}_{ee} = \mathbf{R}_{\mathbf{r} \mathbf{r}} - \mathbf{R}_{\mathbf{r} \mathbf{y}_C} \mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}^{-1} \mathbf{R}_{\mathbf{y}_C \mathbf{r}}. \quad (25)$$

In [17] only the diagonal elements of $\mathbf{R}_{\mathbf{r} \mathbf{r}}$ are used. As we showed in [33] based on the assumption of Gaussian signaling, it is possible to calculate the complete matrix $\mathbf{R}_{\mathbf{r} \mathbf{r}}$, which we will show changes the overall result. We define this operation of a calculating $\mathbf{R}_{\mathbf{r} \mathbf{r}}$ from $\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}$ as $T(\cdot, Q_b(\cdot))$ dependent on the quantization function $Q_b(\cdot)$. Using this definition and (23) in (25) we get:

$$\mathbf{R}_{ee} = T(\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}, Q_b(\cdot)) - (1 - \rho(Q_b(\cdot)))^2 \operatorname{diag}(\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C})^{-\frac{1}{2}} \mathbf{R}_{\mathbf{y}_C \mathbf{y}_C} \operatorname{diag}(\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C})^{-\frac{1}{2}}. \quad (26)$$

Now we can calculate the effective channel $\mathbf{H}'(l)$ and noise covariance matrix $\mathbf{R}_{\eta' \eta'}$ of the overall system including the analog combining and the quantization:

$$\mathbf{H}'(l) = \mathbf{F} \mathbf{W}_R \mathbf{H}(l), \quad (27)$$

and

$$\mathbf{R}_{\eta' \eta'} = \mathbf{F} \mathbf{W}_R \mathbf{R}_{\eta \eta} \mathbf{W}_R^H \mathbf{F}^H + \mathbf{R}_{ee}. \quad (28)$$

It is important to keep in mind that since \mathbf{F} and \mathbf{R}_{ee} are dependent on $\mathbf{R}_{\mathbf{y} \mathbf{y}}$, which depends on $\mathbf{R}_{\mathbf{x} \mathbf{x}}$, the actual effective channel and noise covariance matrix changes with the transmit covariance matrix. Up to now, the actual expression is exact for the case of a Gaussian input signal \mathbf{x} . The actual distribution of e is unknown. Approximating e by a Gaussian with the same covariance matrix leads to simple rate expressions and represent a worst case scenario.

C. Calculation of the receive Covariance matrix

For the calculation of the matrix \mathbf{F} and the effective noise correlation matrix $\mathbf{R}_{\eta' \eta'}$ is necessary to calculate the correlation matrix $\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C}$ of the signal after the analog combining. This signal is defined as:

$$\begin{aligned} \mathbf{y}_C(t) &= \mathbf{W}_R \left(\sum_{l=0}^{L-1} \mathbf{H}(l) \mathbf{x}(t - \tau_l) + \boldsymbol{\eta}(t) \right) \\ &= \mathbf{W}_R (\mathbf{u}(t) + \boldsymbol{\eta}(t)). \end{aligned} \quad (29)$$

Since the two random variables \mathbf{x} and $\boldsymbol{\eta}$ are independent the covariance matrix decomposes into:

$$\mathbf{R}_{\mathbf{y}_C \mathbf{y}_C} = \mathbf{W}_R (\mathbf{R}_{\mathbf{u}(t) \mathbf{u}(t)} + \mathbf{R}_{\boldsymbol{\eta}(t) \boldsymbol{\eta}(t)}) \mathbf{W}_R^H. \quad (30)$$

The things that needs to be calculated is $\mathbf{R}_{\mathbf{u}(t)\mathbf{u}(t)}$.

$$\mathbf{R}_{\mathbf{u}(t)\mathbf{u}(t)} = \mathbb{E} [\mathbf{u}(t)\mathbf{u}^H(t)]. \quad (31)$$

To simplify the notation for the derivation we evaluate the elements of the matrix separately:

$$[\mathbf{R}_{\mathbf{u}(t)\mathbf{u}(t)}]_{i,j} = \mathbb{E} [u_i(t)u_j^*(t)]. \quad (32)$$

Without changing the result, we can transform this equation into the frequency domain and then transform it back. Since the expectation \mathbb{E} and the Fourier transformation \mathcal{F} are linear operations, we can interchange the order we perform them:

$$\begin{aligned} \mathbb{E} [u_i(t)u_j^*(t)] &= \mathcal{F}^{-1} \{ \mathcal{F} \{ \mathbb{E} [u_i(t)u_j^*(t)] \} \} \\ &= \mathcal{F}^{-1} \{ \mathbb{E} [\mathcal{F} \{ u_i(t)u_j^*(t) \}] \}. \end{aligned} \quad (33)$$

With the convolution property of the Fourier transformation we get:

$$\begin{aligned} \mathbb{E} [u_i(t)u_j^*(t)] &= \mathcal{F}^{-1} \{ \mathbb{E} [u_i(f) * u_j^*(-f)] \} \\ &= \mathcal{F}^{-1} \left\{ \mathbb{E} \left[\int_{-\infty}^{\infty} u_i(f')u_j^*(f' - f)df' \right] \right\}. \end{aligned} \quad (34)$$

Due to the linearity of the expectation operation \mathbb{E} we can interchange it with the integrals:

$$\mathbb{E} [u_i(t)u_j^*(t)] = \mathcal{F}^{-1} \left\{ \int_{-\infty}^{\infty} \mathbb{E} [u_i(f')u_j^*(f' - f)] df' \right\}. \quad (35)$$

Since the transmit signal in each frequency bin $\mathbf{x}(f)$ are independent and have zero mean, the expectation $\mathbb{E} [u_i(f')u_j^*(f' - f)]$ is only unequal to zero if $f' = f' - f$. Since outside of the transmission bandwidth the signal is going to be zero, we get the following expression:

$$\mathbb{E} [u_i(t)u_j^*(t)] = \mathcal{F}^{-1} \left\{ \delta(f) \int_{f_1}^{f_2} \mathbb{E} [u_i(f')u_j^*(f')] df' \right\}, \quad (36)$$

with the Dirac function $\delta(f)$ at frequency $f = 0$. So if we transform this Dirac impulse back into the time domain we get:

$$\mathbb{E} [u_i(t)u_j^*(t)] = \int_{f_1}^{f_2} \mathbb{E} [u_i(f)u_j^*(f)] df, \quad (37)$$

independent of time t , that also states that we still have a stationary random process. Plugging the definition of $u_i(f)$ into the equation we get and combining the results to form the covariance matrix $\mathbf{R}_{\mathbf{u}(t)\mathbf{u}(t)}$ we get:

$$\mathbf{R}_{\mathbf{u}(t)\mathbf{u}(t)} = \int_{f_1}^{f_2} \mathbf{H}(f)\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}\mathbf{H}^H(f)df. \quad (38)$$

Combining these results we can express the matrix $\mathbf{R}_{\mathbf{y}_C\mathbf{y}_C}$ as:

$$\begin{aligned} \mathbf{R}_{\mathbf{y}_C\mathbf{y}_C} &= \\ \mathbf{W}_R \left(\int_{f_1}^{f_2} \mathbf{H}(f)\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}\mathbf{H}^H(f)df + \mathbf{R}_{\boldsymbol{\eta}(t)\boldsymbol{\eta}(t)} \right) \mathbf{W}_R^H. \end{aligned} \quad (39)$$

D. Problem Formulation

For the given signal model, the problem of finding the maximum achievable rate for a multipath channel with full Channel State Information (CSI) at the Transmitter (CSIT) and the Receiver (CSIR) can be formulated as:

$$\begin{aligned} R &= \max_{p(\mathbf{x}^N, \mathbf{w}_R^i)} I(\mathbf{x}^N, \mathbf{r}^N | \mathbf{H}(l)) \\ \text{s.t. } &\mathbb{E} [\|\mathbf{x}\|_2^2] \leq P_{Tx} \\ \mathbf{w}_R^i &= [1, e^{j\phi_i}, \dots, e^{j(M_C-1)\phi_i}]^H \quad \forall i \in \{1, \dots, M_{RFE}\}, \end{aligned} \quad (40)$$

with \mathbf{x}^N and \mathbf{r}^N being N input/output samples of the system. Due to the non-linearity of the quantization and the non-trivial problem of finding the optimal beamforming configurations \mathbf{w}_R^i , we make a number of approximations that make the expression traceable:

- Assume \mathbf{x} is Gaussian
- For a system with CSIT SVD based precoding is used (SVD of the effective channel after analog combining is used)
- \mathbf{w}_R^i are selected from the derived finite set separately for each antenna based on a SNR criteria
- Quantization is modeled as an equivalent Gaussian random process

With this simplifications the \mathbf{w}_R^i are already defined and we can transform the problem into a frequency domain equivalent. Since the optimization is carried out for each frequency bin f separate, the result only is a lower bound to the joint optimization.

$$\begin{aligned} R &\leq \int_{f_1}^{f_2} \max_{\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}} I(\mathbf{x}(f), \mathbf{r}(f) | \mathbf{H}'(f)) df \\ \text{s.t. } &\mathbb{E} [\|\mathbf{x}(f)\|_2^2] \leq P_{Tx} \quad \forall f \in [f_1, f_2], \end{aligned} \quad (41)$$

with $\mathbf{x}(f)$, $\mathbf{r}(f)$ and $\mathbf{H}'(f)$ the input/output signal and equivalent channel of frequency bin f . The frequency f_1 and f_2 mark the borders of the band of interest in the equivalent baseband channel. This allows for including oversampling into the expression. If for example not the whole band covered by the sampling rate is available to the system, the parameters f_1 and f_2 have to be set appropriately to account for the oversampling.

Since all signals are represented by Gaussian random variables, we get the following expression for the mutual information:

$$\begin{aligned} &I(\mathbf{x}(f), \mathbf{r}(f) | \mathbf{H}'(f)) = \\ &\log_2 \left(\det \left(\mathbf{I} + \mathbf{R}_{\boldsymbol{\eta}'\boldsymbol{\eta}'}^{-1} \mathbf{H}'(f) \mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \mathbf{H}'^H(f) \right) \right). \end{aligned} \quad (42)$$

For the non-quantized case the result of the optimization is the waterfilling solution. Due to the modeling of the quantization, the effective noise covariance matrix $\mathbf{R}_{\boldsymbol{\eta}'\boldsymbol{\eta}'}$ and the effective channel $\mathbf{H}'(f)$ are dependent on the input covariance matrix $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}$

In a system without quantization, the covariance $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}$ would be chosen according to the right singular vectors of

$\mathbf{H}(f)$ to split the channel in orthogonal subchannels [34]. In this scenario $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}$ would be equal to

$$\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} = \mathbf{V}(f)\mathbf{S}(f)\mathbf{V}^H(f), \quad (43)$$

where $\mathbf{V}(f)$ are the eigenvectors of $\mathbf{H}^H(f)\mathbf{H}(f)$. The diagonal matrix $\mathbf{S}(f)$ represents the power allocation to the subchannels. The optimal allocation follows the waterfilling solution. Since $\mathbf{R}_{\eta'\eta'}$ and $\mathbf{H}'(f)$ actually depend on $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}$, it is difficult to separate the channel into orthogonal subchannels. To make the evaluation traceable, we use the suboptimal precoding vector $\mathbf{V}(f)$. For the matrix $\mathbf{S}(f)$, we test all different possibilities of allocating equal to power to 1 to S_{max} spatial streams. The number S_{max} is the maximum possible number of spatial streams and is equal to $\text{rank}(\mathbf{H}(f))$. If we would allow all frequencies to separately allocate the number of streams, we again have a combinatorial problem. Therefore we check the overall achievable rate for allocating j spatial stream and in the end select one that has the largest achievable rate.

From Equation (28), we see that $\mathbf{R}_{\eta'\eta'}$ is not diagonal. In a system, where the noise covariance matrix is known and independent of the transmit covariance one would simply multiply the receive vector with $\mathbf{R}_{\eta'\eta'}^{-\frac{1}{2}}$. This does generate a new system with a different channel $\mathbf{R}_{\eta'\eta'}^{-\frac{1}{2}}\mathbf{H}'(f)$ and spatial white noise. Then apply the waterfilling solution to the new channel [34]. In general, the achievable rate increases compared to a system with white noise. In a more abstract way, the reason for the improvement is that channels with lower noise power can be used. Dependent on the rank of the channel relative to the number of the receive antennas, the orthogonal subchannels with highest noise power might not be used. In a system where the channel and the noise depend on the covariance matrix of the transmit signal, it is very difficult to generate precoding and reception matrices to split the channel into orthogonal subchannels. Therefore with our system, considering the correlation of the quantization noise leads to a decrease in achievable rate.

For both calculation of the achievable rate in Equation (41) as well as the calculation of the receive signal covariance matrix $\mathbf{R}_{y_C y_C}$ in Equation (39), it is necessary to integrate over the whole signal band from f_1 to f_2 . Instead of taking infinitely small frequency bins, we approximate the signal band in the interval from f_1 to f_2 by a finite number of frequency bins. We choose the number of frequency bins to make the channel $\mathbf{H}(f)$ at each frequency bin sufficiently flat, then the overall achievable rate is well approximated by our result. This reduces Equation (41) to:

$$R \leq \sum_{f_1}^{f_2} \log_2 \left(\det \left(\mathbf{I} + \mathbf{R}_{\eta'\eta'}^{-1} \mathbf{H}'(f) \mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \mathbf{H}'^H(f) \right) \right) \quad \text{s.t. } \mathbb{E}[\|\mathbf{x}(f)\|_2^2] \leq P_{Tx} \quad \forall f \in [f_1, f_2]. \quad (44)$$

Algorithm 2 Approximation of the achievable rate of a quantized system with noise covariance matrix $\mathbf{R}_{\eta\eta}$, multipath channel $\mathbf{H}(l)$ and sum power constraint P_{Tx} and quantization function $Q_b(\cdot)$ with resolution of b bits in the frequency band from f_1 to f_2 .

Require: $\mathbf{R}_{\eta\eta}$, $\mathbf{H}(l)$, P_{Tx} , f_1 , f_2 and $Q_b(\cdot)$

```

1:  $\rho \leftarrow \mathbb{E} [|a - Q_b(a)|^2]$ 
2:  $S_{max} \leftarrow \text{rank} \left( \sum_{l=0}^{L-1} \mathbf{H}(l) \right)$ 
3:  $\mathbf{H}(f) \leftarrow \mathcal{F}(\mathbf{H}(l))$ 
4:  $[\mathbf{V}(f) \ \mathbf{D}(f)] \leftarrow \text{eig}(\mathbf{H}^H(f)\mathbf{H}(f)) \quad \forall f \in [f_1, f_2]$ 
5: for  $j \leftarrow 1$  to  $S_{max}$  do
6:    $\mathbf{S} \leftarrow \mathbf{0}$ 
7:    $[\mathbf{S}]_{i,i} \leftarrow \frac{P_{Tx}}{j} \quad \forall i = \{1, \dots, j\}$ 
8:    $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \leftarrow \mathbf{V}(f)\mathbf{S}\mathbf{V}^H(f) \quad \forall f \in [f_1, f_2]$ 
9:    $\mathbf{R}_{yy} \leftarrow \sum_{f_1}^{f_2} \mathbf{H}(f) \mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \mathbf{H}^H(f) + \mathbf{R}_{\eta(t)\eta(t)}$ 
10:   $\mathbf{R}_{rr} \leftarrow T(\mathbf{R}_{yy}, Q_b(\cdot))$ 
11:   $\mathbf{R}_{ee} \leftarrow \mathbf{R}_{rr} - (1 - \rho)^2 \text{diag}(\mathbf{R}_{yy})^{-\frac{1}{2}} \mathbf{R}_{yy} \text{diag}(\mathbf{R}_{yy})^{-\frac{1}{2}}$ 
12:   $\mathbf{F} \leftarrow (1 - \rho) \text{diag}(\mathbf{R}_{yy})^{-\frac{1}{2}}$ 
13:   $\mathbf{R}_{\eta'\eta'} \leftarrow \mathbf{R}_{ee} + \mathbf{F} \mathbf{R}_{\eta\eta} \mathbf{F}^H$ 
14:   $\mathbf{H}'(l) \leftarrow \mathbf{F} \mathbf{H}(l) \quad \forall l \in \{0, \dots, L-1\}$ 
15:   $\mathbf{H}'(f) \leftarrow \mathcal{F}(\mathbf{H}'(l))$ 
16:   $\mathbf{A}(f) \leftarrow \mathbf{I} + \mathbf{R}_{\eta'\eta'}^{-1} \mathbf{H}'(f) \mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \mathbf{H}'^H(f) \quad \forall f \in [f_1, f_2]$ 
17:   $R(j) = \sum_{f_1}^{f_2} \log_2 (\det(\mathbf{A}(f)))$ 
18: end for
19:  $R_{max} \leftarrow \max_j R(j)$ 
20: return  $R_{max}$ 
```

Receiver correlation matrix can then be calculated as:

$$\mathbf{R}_{y_C y_C} = \mathbf{W}_R \left(\sum_{f_1}^{f_2} \mathbf{H}(f) \mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)} \mathbf{H}^H(f) + \mathbf{R}_{\eta(t)\eta(t)} \right) \mathbf{W}_R^H. \quad (45)$$

The channel $\mathbf{H}(f)$ or the effective channel $\mathbf{H}'(f)$ at the frequency bins f can be calculated from the channel taps $\mathbf{H}(l)$ via the DFT $\mathcal{F}(\cdot)$:

$$\mathbf{H}(f) = \mathcal{F}(\mathbf{H}(l)). \quad (46)$$

We now have all the necessary mathematical tools to approximate the achievable rate of a multipath channel including quantization effects at the receiver. Algorithm 2 describes our approximation of the achievable rate for these type of systems.

This approximation is modeling a point to point closed loop spatial multiplexing system. There are many different simple modification possible to change the modeled system. The following are a non-exhaustive list of examples:

- Systems without CSIT
- Systems with imperfect channel estimation
- Systems with multiple terminals communication with base station

- Systems with constraint feedback
- Systems with multiple terminals and a basestation

Most of these systems can be modeled by changing the selection of the constraints of the precoding matrix $\mathbf{R}_{\mathbf{x}(f)\mathbf{x}(f)}$ and the channel model.

IV. EVALUATION RESULTS

In this section we evaluate the derived expression for different scenarios. As comparison, we always include a rate evaluation without quantization. We use a system that applies the waterfilling solution separate for each frequency bin. For all scenarios the results show the average achievable rate in bps/Hz averaged over 1000 channel realizations.

A. Comparison to diagonal approximation

This part of the evaluation compares difference in performance when considering the non-diagonal elements in the calculation of $\mathbf{R}_{\mathbf{r}\mathbf{r}}$ and therefore $\mathbf{R}_{\mathbf{r}\mathbf{r}'}\mathbf{r}'$. The model considering only the diagonal elements of $\mathbf{R}_{\mathbf{r}\mathbf{r}}$ was used in [19] and [17].

For the evaluation a channel of the first channel model is used. Here $K = 7$ separate path are received at the same time. Different number of transmit and receive antennas are used. From Figure 6, we see that the model considering the off diagonal elements (ND) has a significant lower performance compared to the model only considering the diagonal elements (D). In fact, for the case of only one transmit antenna ($M_t = 1$) for 1-3 bit A/D conversion, the achievable rate is not maximized at the highest SNR possible, but rather at a finite SNR between 0 and 10 dB.

As discussed in section III, if we compare the results in Figure 6 to the ones in Figure 7. Considering the off diagonal elements has only a large influence if the number of receive antennas is larger than the number of transmit antennas. This effect can be explained in the following ways: After spatial whitening, the power distribution of the effective noise is more non-uniform relative to the system that considers only the diagonal component. Since the actual channel and noise covariance matrix depends on the precoding matrix, it is not possible to decompose the channel into orthogonal subchannels. Therefore we cannot avoid using the channel with high noise variance and therefore the overall performance does decrease.

Another important thing to mention is that in a system with multipath propagation and white noise, the covariance matrix $\mathbf{R}_{\mathbf{y}\mathbf{y}}$ of the receive signal is approximated diagonal. This leads to a diagonal matrix $\mathbf{R}_{\mathbf{r}\mathbf{r}}$ and therefore spatial white noise of the quantized system.

B. Influence of AGC imperfection

In this evaluation, we show the influence of AGC imperfections on the performance. To limit the evaluation to show the influence of an imperfect adapted AGC, we chose a SISO system with the simple multipath model described in the signal model with the parameters $L = 32$, $P = 16$ and $\beta = 0.35$. For an imperfect AGC, the assumption that the receive signal \mathbf{r} and the quantization error \mathbf{e} are independent is no longer satisfied.

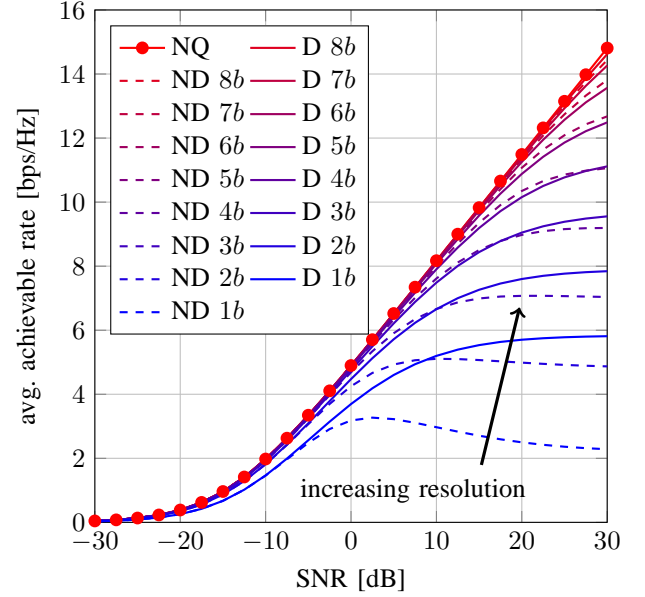


Fig. 6. $M_R = 32$ and $M_T = 1$ digital beamforming with different resolution of the ADC.

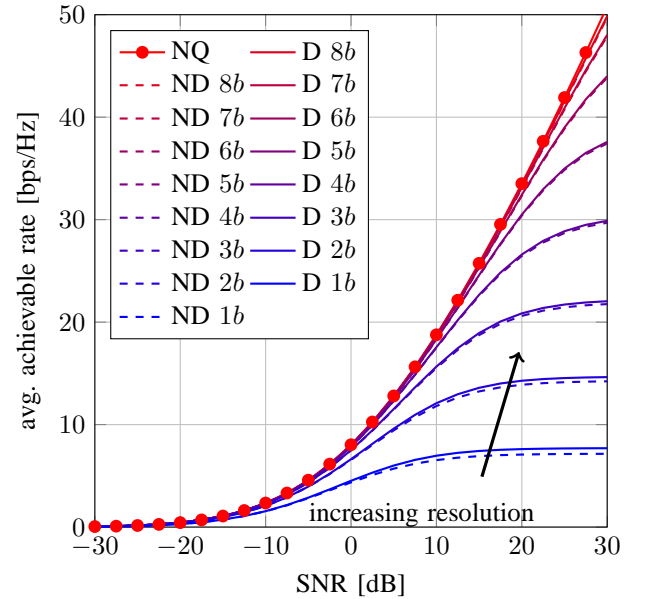


Fig. 7. $M_R = 8$ and $M_T = 8$ digital beamforming with different resolution of the ADC.

Since all our formulas for modeling the quantization are based on this assumption $\mathbb{E}[r_i e_i^H] = 0 \forall i = \{1, \dots, M_R\}$, they are no longer valid in the case of an imperfect adapted AGC. We can enforce this orthogonality by scaling the signal \mathbf{r} . The scaling factor α is equal to:

$$\alpha = \frac{\mathbb{E}[a Q_b(a)]}{\mathbb{E}[Q_b(a)^2]}, \quad (47)$$

with a being a real Gaussian random variable with unit variance and zero mean. After this scaling we can use the derived formulas as before.

The graphs in Figure 8 show the average achievable rate

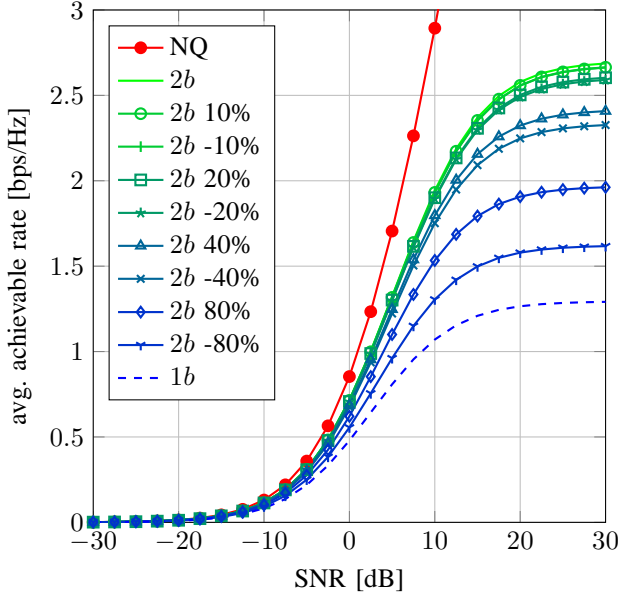


Fig. 8. SISO system with imperfect AGC and 2bit ADC resolution.

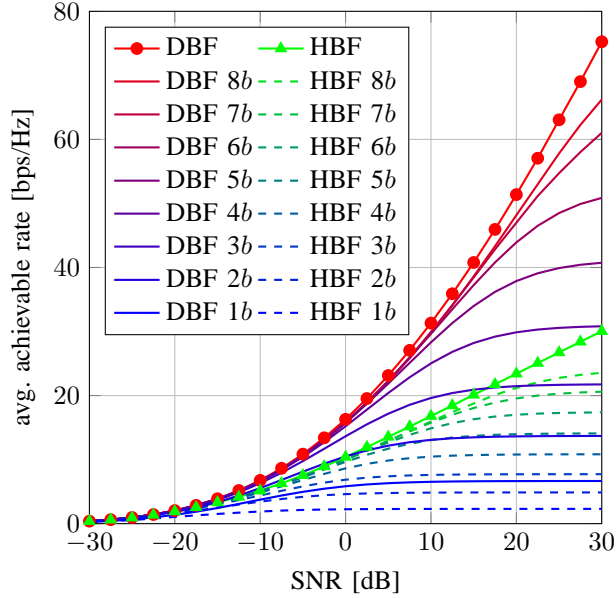


Fig. 9. $M_R = 8$ and $M_T = 64$ $M_C = 4$ different resolution of the ADC b .

with 2 bit resolution and different offset relative to the optimal power at the VGA output. The Graphs shows that an error in the range of -20% to 20% has only a minor impact on the performance, but as soon as the error is larger than that, the performance decreases dramatically. Ultimately, the quantization converges to 1-bit quantization and therefore also our achievable rate converges to the one of 1-bit quantization. We can also observe the performance penalty for a larger negative or positive error is different.

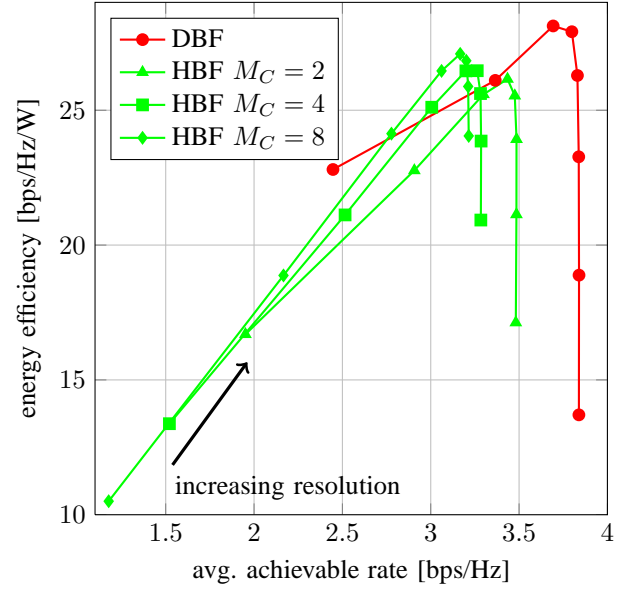


Fig. 10. $M_R = 8$ and $M_T = 64$ SNR -15dB different resolution of the ADC b .

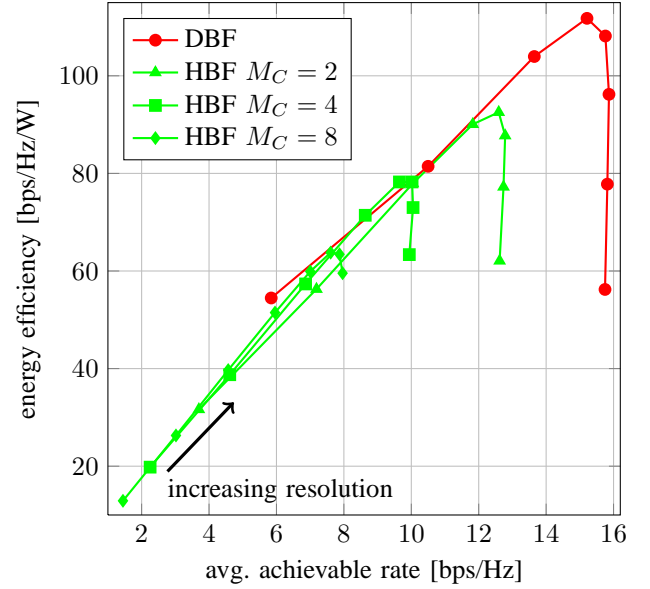


Fig. 11. $M_R = 8$ and $M_T = 64$ SNR 0dB different resolution of the ADC b .

C. Downlink (DL) Point to point scenario

In this subsection, a downlink like scenario is evaluated. A basestation with 64 antennas ($M_T = 64$) is transmitting to a mobilestation with 8 antennas ($M_R = 8$). For the channel model the following parameters are used: $L = 32$, $P = 16$, $\beta = 0.35$. For the hybrid beamforming system $M_C \in \{2, 4, 8\}$ and therefore, $M_{RFE} \in \{4, 2, 1\}$ are used.

Figure 9 show the average achievable rate for the case of $M_C = 4$ and ADC resolution $b \in \{1, \dots, 8\}$. The rate curves of the systems including an ADC clearly converge to the ones assuming no ADC for higher resolution in both cases of hybrid and digital beamforming. Especially in the low SNR regime

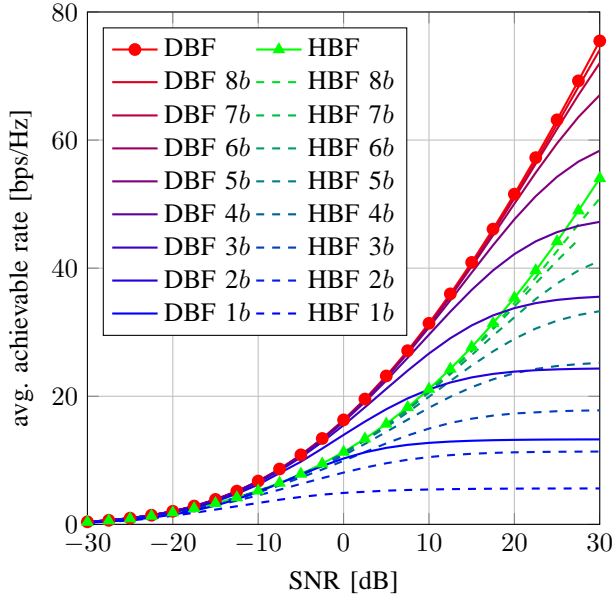


Fig. 12. $M_R = 64$ and $M_T = 8$ $M_C = 4$ different resolution of the ADC b .

(below 0 dB), the performance of the digital beamforming systems with low resolution ADC (1-3 bit) are very close to the performance without considering an ADC. These systems clearly outperform a hybrid beamforming system in this SNR regime. In this evaluation a 4-bit ADC is enough to outperform the hybrid system over the whole SNR range.

Since these system have a different power consumption, we also have to compare the results in terms of energy efficiency. Here we define the energy efficiency (EE) as the average achievable rate R divided by the power consumption of the RF-front-end P_R :

$$EE = \frac{R}{P_R}. \quad (48)$$

Figure 10 and 11 show the energy efficiency over the achievable rate for $M_C \in \{2, 4, 8\}$ and the resolution of the ADC $b \in \{1, \dots, 8\}$ with $SNR \in \{-15\text{dB}, 0\text{dB}\}$. For both cases, the digital beamforming achieves a higher data rate and also a higher spectral efficiency. In the -15dB SNR, case the difference in energy efficiency is not substantial but in the 0dB SNR there is a large gap between hybrid and digital beamforming. In the lower SNR case, the energy efficiency peaks at 3-bit ADC resolution. The higher the SNR gets, the larger the ADC resolution that maximizes the energy efficiency. These results shows that even when perfect hybrid beamforming without the beam-alignment overhead is considered a digital beamforming system is more energy efficient.

D. Uplink (UL) Point to point scenario

For the configuration of the system, the same parameters as in the DL like setup in the previous subsection are used. The only difference is that in this case the antenna configuration is $M_R = 64$ and $M_T = 8$.

Figure 12 shows the achievable rate for this cases. We observe that the penalty of hybrid beamforming is less severe

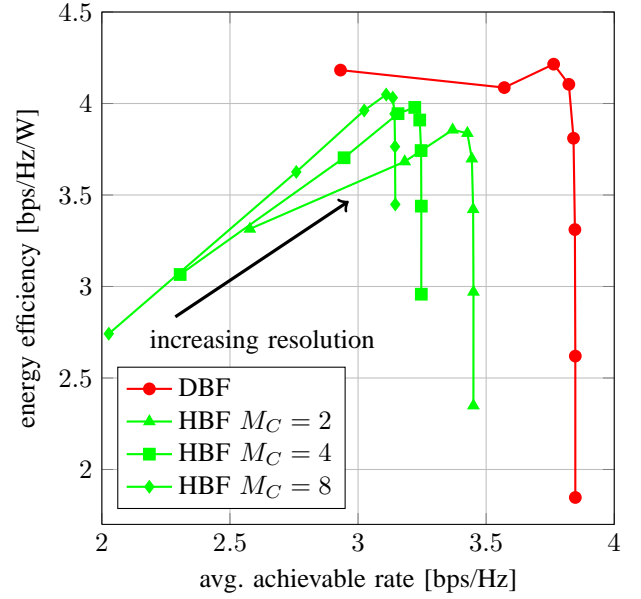


Fig. 13. $M_R = 64$ and $M_T = 8$ SNR -15 dB different resolution of the ADC b .

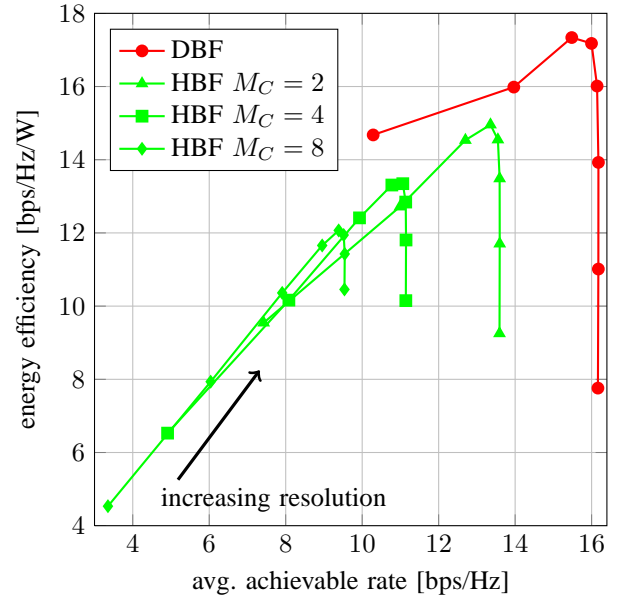


Fig. 14. $M_R = 64$ and $M_T = 8$ SNR 0 dB different resolution of the ADC b .

than in the DL case. The reason is that in this case, the side of the system with less antennas (the mobile station) has no constraints on the front-end which is the exact opposite in the DL. This means that the number of spatial streams is in all cases just limited by the 8 possible streams of the mobile station. Therefore, the penalty of hybrid beamforming is less and the achievable rate of hybrid and digital beamforming rise with the same slope for the case without quantization.

In the low to medium SNR, the low resolution ADC digital beamforming systems perform better than the hybrid beamforming one. In the high SNR regime, there is no penalty

on the number of possible data streams for the hybrid systems, therefore it performs better in this regime.

Figure 13 and 14 show the energy efficiency over the achievable rate for $M_C \in \{2, 4, 8\}$ and the resolution of the ADC $b \in \{1, \dots, 8\}$ with $\text{SNR} \in \{-15\text{dB}, 0\text{dB}\}$. As in the DL the digital beamforming system is more energy efficient as well as achieving a higher rate. Due to the small number of antennas, the energy efficiency stays almost constant for 1 to 3 bit ADC resolution. This can be explained with the fact that if the resolution is small the power consumption of the front-end is dominated by the other components, and the fact that we have a large degree of freedom with 64 Antennas and therefore the influence of the quantization noise at each of the antennas is not very significant.

V. CONCLUSION

The evaluation in this paper showed that low resolution ADC digital beamforming systems are more energy efficient and achieving a higher rate than hybrid beamforming systems for the given scenarios, especially in the low to medium SNR region the performance. We also showed that if the imperfections of the AGC is in the range of -20% to 20%, there is no major influence on the performance. The evaluation of including the off diagonal elements in the quantization error model showed that this could have a substantial impact on the performance with very low resolution ADC.

Future extensions should consider the following points. For the hybrid beamforming case, the evaluation only shows the result if the beams are already aligned. As shown in [6], can be considered to be a large overhead. A possible future mobile broadband system operating at mmWave frequencies will definitely suffer from additional other RF-frontend related constraints. Especially considering the inefficiency of the PA that have to operate close to the saturation and therefore, introduce non-linear distortion of the signal. Also phase noise scales approximately with carrier frequency squared and thus, has to be considered for mmWave systems. This will lead to a limited constellation size, which will then bound the overall spectral efficiency. In this evaluation, we also ignored the necessary reference overhead for channel estimation. Especially for a high order spatial multiplexing, this is not negligible and will essentially limit the number of spatial data streams. The channel model is assuming a omnidirectional minimum scattering antenna. Including this consideration into the evaluation would lead to a result that is more close to a practical evaluation. A dynamic multi-user environment would also provide for an interesting comparison between hybrid and low resolution ADC digital beamforming.

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